

Characterizing Collisions

From Bounce Times to Velocity, Energy, and the Coefficient of Restitution

A step-by-step analysis guide • Scientific Inquiry with AI

In this experiment you record the **sound** of a ping-pong ball bouncing on a hard surface. Each impact makes a sharp click, so the audio gives you the precise **clock time of every bounce**. From this one kind of measurement — a list of bounce times — you can reconstruct the ball's **speed**, the **height** it reaches, the **energy** it loses in each collision, and the **coefficient of restitution** that characterizes how “bouncy” the collision is.

This guide builds the analysis up one step at a time, starting from the equations of motion you already know. Nothing is skipped: every formula is derived from the one before it, and worked numbers are shown so you can check your own work.

What you measure vs. what you calculate

You **measure** only the bounce times $t_1, t_2, t_3, \dots$. Everything else — velocity, height, energy, and the coefficient of restitution — is **calculated** from those times using the physics below.

1. The Raw Measurement: Bounce Times and the Time Between Bounces

The ball is dropped from an initial height H_0 . It strikes the surface, rebounds to a lower height, falls, strikes again, and so on. We label the bounces with an index $n = 1, 2, 3, \dots$ and call the clock time of the n -th impact t_n .

The quantity that carries the physics is the **time between successive bounces**, which we call the **flight interval** T_n :

$$T_n = t_{n+1} - t_n \quad (1)$$

In words: T_n is how long the ball is in the air between bounce n and bounce $n+1$. During that interval the ball leaves the floor, rises to a peak height h_n , and falls back down. Figure 1 shows these quantities.

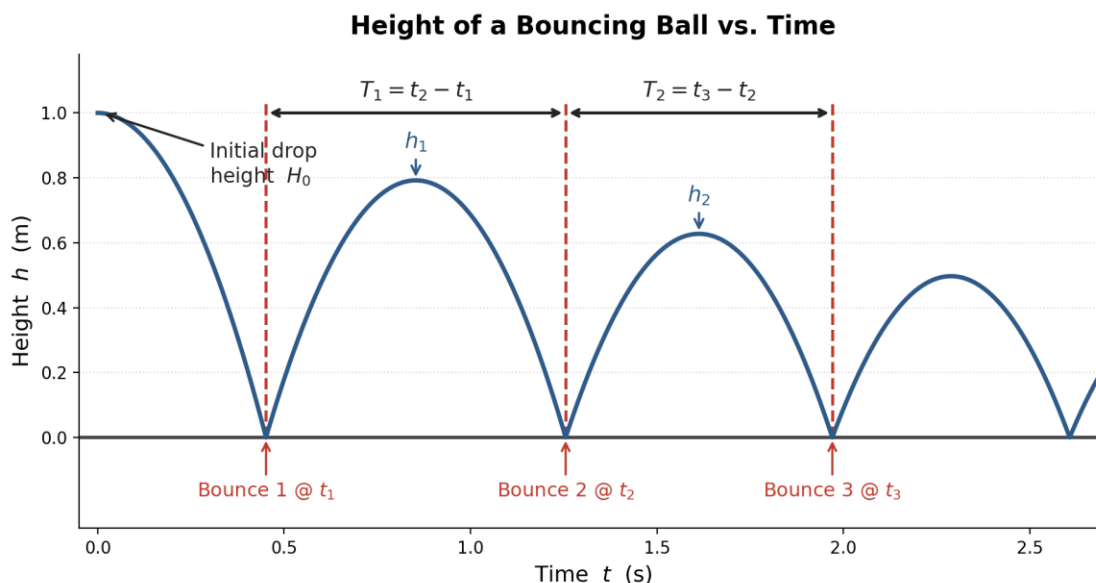


Figure 1. Height vs. time. Each red dashed line marks a bounce time t_n . The flight interval T_n is the time between two bounces; h_n is the peak height the ball reaches during that interval. The peaks shrink because energy is lost at each collision.

Two words that are easy to confuse

Bounce time: t_n = the moment an impact happens (read directly from the audio).

Flight interval: T_n = the gap of time between two impacts (a difference of two bounce times).

All of the analysis below uses the intervals T_n .

2. Quick Review: Equations of Motion for Constant Acceleration

While the ball is in the air, the only force on it is gravity (for now we assume no drag force), so it moves with a **constant acceleration**. For any object with constant acceleration a , starting at position y_0 with velocity v_0 , the three standard kinematic equations are:

$$v = v_0 + a t \quad (2)$$

$$\Delta y = v_0 t + \frac{1}{2} a t^2 \quad (3)$$

$$v^2 = v_0^2 + 2 a \Delta y \quad (4)$$

Here v is the velocity at time t , and Δy is the change in height. For our bouncing ball we take **upward as positive**, so the acceleration is downward: $a = -g$, with $g = 9.81 \text{ m/s}^2$ (the magnitude of free-fall acceleration). These three equations are the only physics we need; the rest of the guide is algebra applied to them.

3. From a Flight Interval to Speed and Height

Consider one trip through the air: the ball leaves the floor going straight up with some **rebound speed** w_n , rises, slows to a stop at the top, then falls back. Because gravity is the same on the way up and the way down, the motion is **symmetric**: the time to rise equals the time to fall. So the ball reaches its peak exactly halfway through the interval, at time $\frac{T_n}{2}$.

3a. Rebound speed from the flight interval

At the very top the vertical velocity is zero. Apply Equation (2) from launch (velocity w_n) to the apex (velocity 0), over the rise time $\frac{T_n}{2}$:

$$0 = w_n - g \frac{T_n}{2} \quad (5)$$

Solving for the rebound speed gives a clean result — the speed the ball left the floor with is set entirely by how long it stays in the air:

$$w_n = \frac{1}{2} g T_n \quad (6)$$

Worked example. If a flight interval is $T_n = 0.40 \text{ s}$, then $w_n = \frac{1}{2}(9.81)(0.40) = 1.96 \text{ m/s}$.

3b. Peak height from the flight interval

The ball rises for a time $\frac{T_n}{2}$. Using Equation (3) for the rise (starting speed w_n , acceleration $-g$), the peak height is

$$h_n = w_n \frac{T_n}{2} - \frac{1}{2} g \frac{T_n^2}{2} \quad (7)$$

Substitute $w_n = \frac{1}{2} g T_n$ from Equation (6) and simplify. The two terms combine to give the compact form:

$$h_n = \frac{1}{8} g T_n^2 \quad (8)$$

(Equivalently, since $h_n = \frac{w_n^2}{2g}$ from Equation (4) with $v = 0$ at the top — both routes give the same answer.)

Worked example. With $T_n = 0.40 \text{ s}$: $h_n = \frac{1}{8}(9.81)(0.40)^2 = 0.196 \text{ m}$.

4. Energy of the Bounce

Just after leaving the floor, all of the ball's mechanical energy is **kinetic**. At the top of its flight, all of it is **potential**. Energy conservation in the air means these are equal, so the mechanical energy associated with flight interval n can be written either way:

$$E_n = \frac{1}{2} m w_n^2 = m g h_n \quad (9)$$

Because both the kinetic energy ($\propto w^2$) and the potential energy ($\propto h$) are fixed by the flight interval, you do not even need the ball's mass to compare one bounce to the next — the **ratios** of energy depend only on the measured times. That is exactly what the coefficient of restitution captures.

5. The Coefficient of Restitution, e

The **coefficient of restitution** e measures how much speed survives a collision. It is defined as the ratio of the speed leaving the surface to the speed arriving at it:

$$e = \frac{\text{rebound speed}}{\text{approach speed}} \quad (10)$$

A perfectly elastic bounce has $e = 1$ (no speed lost); a dead, fully inelastic impact has $e = 0$. Real balls fall in between. There are two complementary ways to get e from your bounce times.

5a. Method 1 — e for each bounce, from approach and rebound speeds

Here is the key insight that links speeds to your measured intervals. When the ball comes back down to the floor, it arrives at the **same speed it left with** (the flight is symmetric). So the speed approaching bounce n is just the rebound speed from the *previous* interval, w_{n-1} , and the speed leaving bounce n is w_n . Therefore:

$$e_n = \frac{w_n}{w_{n-1}} \quad (11)$$

Now substitute the speed–interval relation $w_n = \frac{1}{2} g T_n$ from Equation (6). The factor of $\frac{1}{2} g$ cancels top and bottom, leaving a remarkably simple result:

$$e_n = \frac{\frac{1}{2} g T_n}{\frac{1}{2} g T_{n-1}} = \frac{T_n}{T_{n-1}} \quad (12)$$

Result

The coefficient of restitution for each bounce is simply the **ratio of consecutive flight intervals**. You can read e straight off the bounce times — no mass, no height, no calibration required:

$$e_n = T_n / T_{n-1}.$$

5b. Energy lost per collision

Since energy goes as speed squared (Equation 9), the fraction of kinetic energy **kept** in a bounce is e_n^2 , and the fraction **lost** is whatever is left over:

$$\text{fraction of energy lost} = 1 - e_n^2 = 1 - \frac{T_n^2}{T_{n-1}^2} \quad (13)$$

Worked example. If $e_n = 0.96$, the ball keeps $(0.96)^2 = 0.92$, i.e. 92% of its energy, and loses about 8% in that single collision.

5c. Relating e to the rebound heights

You can also express e through the peak heights, but be careful: height scales with speed **squared**, not speed itself. Since $h_n \propto w_n^2$, the height ratio is the *square* of the speed ratio:

$$\frac{h_n}{h_{n-1}} = \frac{w_n^2}{w_{n-1}^2} = e_n^2 \quad (14)$$

so taking the square root gives the coefficient of restitution from heights:

$$e_n = \sqrt{\frac{h_n}{h_{n-1}}} \quad (15)$$

Watch the square root

A common slip is to write $e = h_n / h_{n-1}$ without the square root. That would be the energy ratio, not the speed ratio. The speed-based definition of e requires the square root, because **energy and height scale as speed squared**.

6. The Best Estimate of e : The Log Plot Method

Method 1 gives a value of e from every neighboring pair of bounces, but each one is a bit noisy. If the surface and ball behave the same way at every bounce, then e is roughly **constant**, and we can use **all** the bounces at once to pin it down far more precisely. Here is how.

If e is constant, each interval is just e times the one before it (from Equation 12, $T_n = e \cdot T_{n-1}$). Applying that repeatedly:

$$T_2 = e T_1, T_3 = e T_2 = e^2 T_1, \dots$$

which gives a **geometric pattern** — the intervals shrink by the same factor each time:

$$T_n = T_1 e^{n-1} \quad (16)$$

Geometric patterns become **straight lines** when you take a logarithm. Taking the natural log of both sides of Equation (16) and using $\ln(A B) = \ln A + \ln B$ and $\ln(e^k) = k \cdot \ln e$:

$$\ln T_n = \ln T_1 + (n - 1) \ln e$$

Grouping the terms by powers of n puts this in the form of a straight line, $y = (\text{slope}) \cdot n + (\text{intercept})$:

$$\ln T_n = (\ln e) n + (\ln T_1 - \ln e) \quad (17)$$

So if you plot $\ln T_n$ on the vertical axis against the bounce number n on the horizontal axis, the data fall on a straight line whose **slope equals $\ln e$** . Figure 2 shows this.

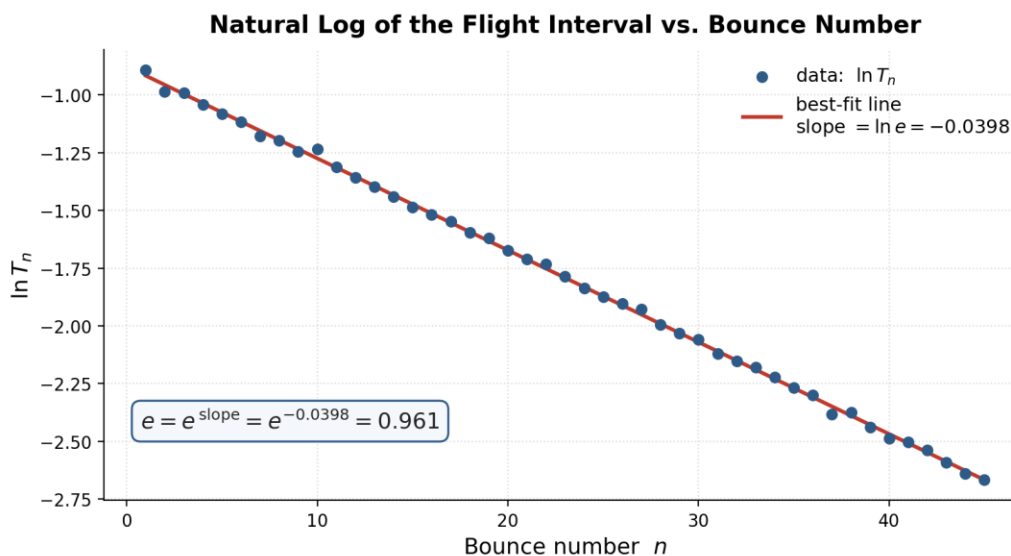


Figure 2. Plotting $\ln T_n$ against bounce number n gives a straight line. Fitting the line gives one precise slope from all the data at once; the coefficient of restitution is then $e = e^{\text{slope}}$.

To recover e itself, undo the logarithm by exponentiating the slope:

$$e = e^{\text{slope}} \quad (18)$$

Worked example (from real bounce data)

A line fit to bounces 1–45 gives a slope of -0.0398 . Then $e = e^{-0.0398} = 0.961$. This single number summarizes all 45 bounces and is far less sensitive to noise in any one bounce than the pair-by-pair values from Method 1.

A note on which logarithm. The method works with any base of logarithm. If you use base-10 instead of natural log, you determine e with $e = 10^{\text{slope}}$. The data here were fit with the natural log, so we use $e = e^{\text{slope}}$.

7. Putting It Together: From Times to Physics

Every quantity in this experiment flows from the single list of bounce times. The chain of reasoning is:

- **Measure** the bounce times t_n from the audio.
- **Subtract** to get the flight intervals $T_n = t_{n+1} - t_n$.
- **Compute** rebound speed w_n , peak height h_n , and energy E_n for each interval.
- **Characterize** the collisions with e — either bounce-by-bounce (ratio of intervals) or, more precisely, from the slope of the log plot.
- **Interpret:** the fraction of energy lost per collision is $1 - e^2$.

Quick-reference equation sheet

Quantity	Formula (from bounce times)
Flight interval	$T_n = t_{n+1} - t_n$
Rebound speed	$w_n = \frac{1}{2} g T_n$
Peak height	$h_n = \frac{1}{8} g T_n^2$
Mechanical energy	$E_n = \frac{1}{2} m w_n^2 = m g h_n$
COR (per bounce)	$e_n = \frac{T_n}{T_{n-1}} = \sqrt{\frac{h_n}{h_{n-1}}}$
COR (log-plot slope)	$\text{slope of } \ln T_n \text{ vs. } n = \ln e; e = e^{\text{slope}}$
Energy lost / collision	$1 - e_n^2$

Every equation above is a native Word equation — click any of them to edit in the equation editor.