

Latitude and longitude

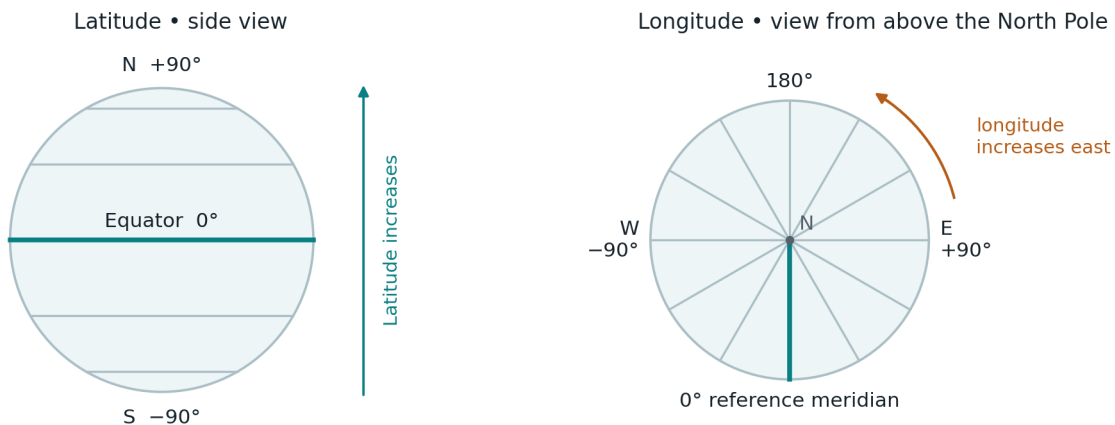
Two angles describe a place

Your phone reports its location as coordinates. This reading helps you interpret those numbers, infer the direction of a walk, and estimate short distances in meters. Read pages 1–4 and complete the nine worksheet questions. Allow about 25–30 minutes for both; the math appendix is optional.

Start with the reference lines

Latitude is angular position north or south of the equator. The equator is 0° ; signed decimal latitude is positive north and negative south. The poles are $+90^\circ$ and -90° .

Longitude is angular position east or west of the zero-longitude reference meridian near Greenwich. In the signed convention used here, east is positive and west is negative, from -180° to $+180^\circ$. All meridians meet at the poles, so longitude is undefined there.



The diagrams use a spherical Earth. Lines of constant latitude run east-west; you cross them when moving north-south. Longitude meridians run from pole to pole; you cross them when moving east-west. Seen from above the North Pole, longitude increases toward the east.

Read the order and the units

We write coordinate pairs as (latitude, longitude), in decimal degrees. For example, $(37.36000^\circ, -120.43000^\circ)$ is north of the equator and west of Greenwich. Some software reverses the order, so always check the column labels.

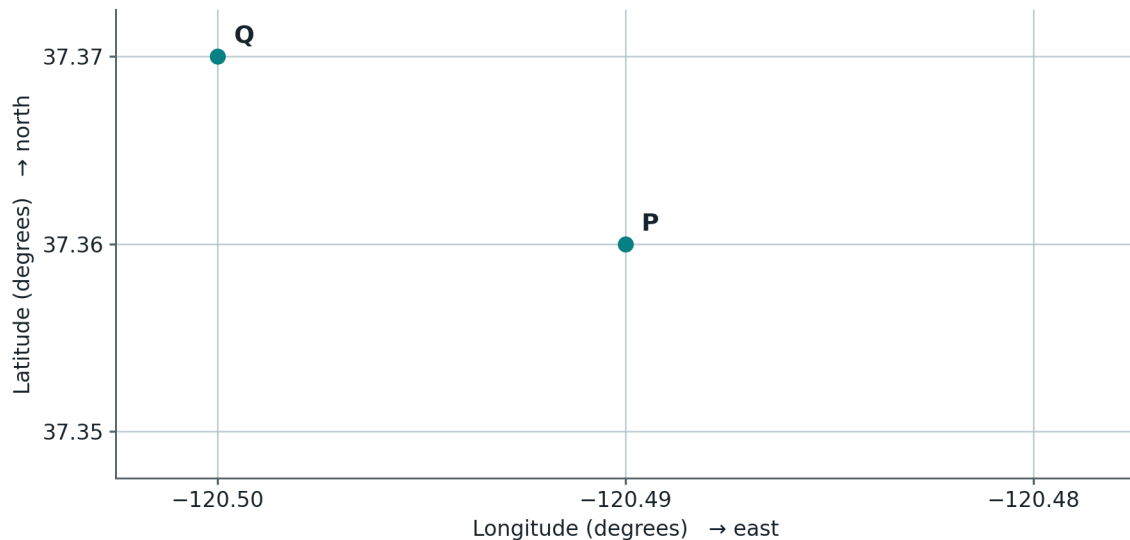
A phone reports these coordinates in the WGS 84 reference frame, the same one GPS uses.

Pause and explain: Why can a line of latitude run east-west even though latitude tells you about north-south position?

Read coordinates on a grid

Increasing numbers reveal direction

On this north-up grid, latitude is the vertical coordinate and longitude is the horizontal coordinate. Read the axis labels before interpreting a point. This grid is for coordinate practice; equal steps in degrees need not represent equal distances in meters.



Use final minus initial

A coordinate change is the final value minus the initial value. An increase in latitude means northward motion; a decrease means southward motion. Locally, an increase in longitude means eastward motion; a decrease means westward motion.

Negative longitude can increase while it remains negative. Moving from -120.50° to -120.49° gives $\Delta\text{longitude} = -120.49^\circ - (-120.50^\circ) = +0.01^\circ$. That is an eastward change. “West of Greenwich” describes location; “moving east” describes a change in location.

Combine the two changes

If latitude and longitude both increase, the displacement has northward and eastward components. If only latitude changes, the idealized motion is north-south. If neither coordinate changes, the idealized position is constant.

For this lesson, compare nearby points around campus. A route crossing the $\pm 180^\circ$ longitude boundary needs special handling; ordinary subtraction can give a misleading large change.

Try it: Read P and Q as (latitude, longitude). If you move from P to Q, what happens to each coordinate?

Convert coordinate changes to meters

Use a local scale for each coordinate

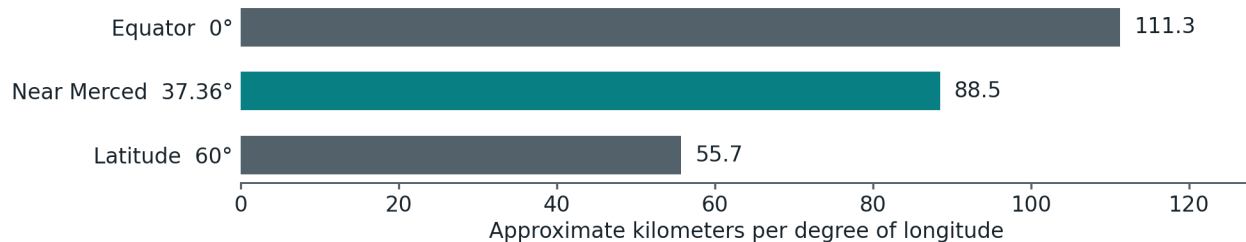
Degrees measure angles. To estimate a walking distance, multiply a coordinate change by a local conversion factor. Near Merced, at latitude about 37.36° , use these classroom approximations consistently:

Northward change (m) $\approx \Delta\text{latitude (degrees)} \times 111,000 \text{ m/degree}$

Eastward change (m) $\approx \Delta\text{longitude (degrees)} \times 88,500 \text{ m/degree}$

Why are the factors different

Longitude meridians meet at the poles. A fixed angular longitude change therefore spans a smaller surface distance at higher latitudes. One degree of latitude remains roughly 111 km; one degree of longitude depends strongly on latitude.



Spherical-model values before rounding. The highlighted bar is the latitude of this lesson. At the equator a degree of longitude spans about the same distance as a degree of latitude, which is where the 111,000 m/degree latitude factor comes from.

A worked example

A student moves from $(37.36000^\circ, -120.43000^\circ)$ to $(37.36020^\circ, -120.42970^\circ)$. Subtract before converting:

$\Delta\text{latitude} = +0.00020^\circ \rightarrow +22.2 \text{ m north.}$

$\Delta\text{longitude} = +0.00030^\circ \rightarrow +26.55 \text{ m east.}$

The approximate straight-line separation is $\sqrt{(22.2^2 + 26.55^2)} \approx 34.6 \text{ m}$. This is the separation of the endpoints. It is the route length only if the route between them is approximately straight.

Keep the signs to express direction: a negative northward change means south, and a negative eastward change means west.

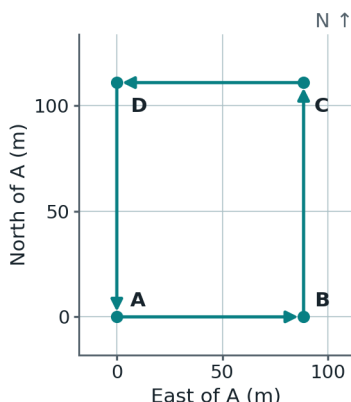
Then report at the precision the data support. These coordinate changes carry two digits, so the answer is about 22 m north, 27 m east, and a 35 m separation. Carrying extra digits through the arithmetic protects against round-off; it does not create measurement accuracy.

Optional: pages 5–7 derive these factors and check how accurate they are.

Use coordinates for a campus walk

Connect the numbers to a route

This invented practice route is near Merced in coordinate value. It is a schematic, not a verified campus walkway. Both axes of the sketch use the same scale. Travel $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$.



Waypoint	Latitude (°)	Longitude (°)
A	37.36000	-120.43000
B	37.36000	-120.42900
C	37.36100	-120.42900
D	37.36100	-120.43000

Notice which coordinate remains constant on each leg. Compute each leg separately, then add the leg lengths to estimate the total distance traveled. This route ends where it began. The change from the start to the final point describes displacement; it does not record every part of the journey.

Keep the measurement in view

A phone reports estimated coordinates. Small coordinate changes while it lies still reflect measurement variation rather than real walking, and stable digits do not by themselves prove that the reported location is accurate. Coordinate precision and location accuracy are different: extra decimal places make smaller numerical changes representable, but they do not mean the receiver located you that closely.

Most sensor apps also report a horizontal accuracy in meters beside each position. That column, not the number of decimal places, is the receiver's own uncertainty estimate, and for a phone it is usually a few meters. Record it, and use a stationary trial and known route features as checks during the lab.

Complete the worksheet before asking for full solutions. Preserve your first answers and add revisions so you can explain what changed in your reasoning.

Coordinate references: NOAA, What is latitude? and What is longitude? Those pages describe latitude and longitude as distances north-south and east-west; this reading treats them as angles first and converts to meters separately.

<https://oceanservice.noaa.gov/facts/latitude.html>

<https://oceanservice.noaa.gov/facts/longitude.html>

Optional mathematics appendix

From a circle to a latitude factor

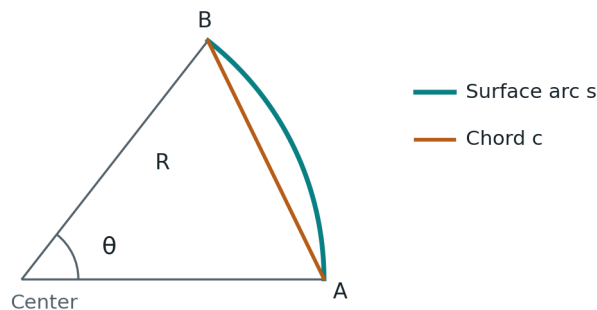
You can complete the worksheet without this appendix. Here we model Earth as a sphere with radius $R \approx 6,378,000$ m, approximately its equatorial radius. Earth is slightly flattened; this is a classroom model, not a surveying formula.

Convert degrees to radians

One full turn is 360° or 2π radians. Radians express an angle as arc length divided by radius:

θ in radians = θ in degrees $\times \pi/180$

Arc length $s = R\theta$ when θ is in radians.



The sketch exaggerates the angle so you can see the curved surface arc and the straight chord through the sphere.

Identify the approximation

For a small angle u measured in radians, $\sin(u) \approx u$. The chord length is therefore:

$c = 2R \sin(\theta/2) \approx 2R(\theta/2) = R\theta = s$.

The arc relation $s = R\theta$ is exact on a circle. The approximation is replacing the curved arc with the straight chord, which is the same as treating a short piece of the surface as flat. For a 100 m arc, $\theta = 100/6,378,000 \approx 0.0000157$ radians, and the chord falls short of the arc by about a nanometer.

Calculate the latitude factor

North-south travel follows a meridian circle of radius R . For a 1° change:

$R \times \pi/180 \approx 111,317$ m per degree.

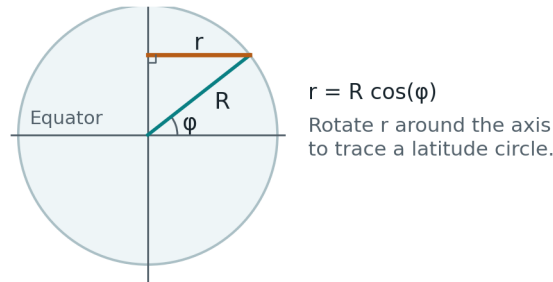
This lesson uses **111,000 m per degree of latitude**.

The spherical illustration uses geocentric latitude. Precise GPS coordinates use geodetic latitude on an ellipsoid; page 6 checks what the spherical model and the rounding cost.

Optional mathematics appendix

Longitude and the limits of a local map

At latitude ϕ (phi), the east-west latitude circle is smaller than the equatorial circle. A right triangle gives its radius r :



$\cos(\phi) = r/R$, so $r = R \cos(\phi)$. Using this radius in the arc-length equation gives meters per degree of longitude:

$R \cos(\phi) \times \pi/180$.

Near Merced, $\phi \approx 37.36^\circ$ and $\cos(\phi) \approx 0.79484$:

$6,378,000 \times 0.79484 \times \pi/180 \approx 88,479$ m per degree.

This lesson uses **88,500 m per degree of longitude**. Set your calculator to degrees when evaluating $\cos(37.36^\circ)$.

How good are these factors

Keep extra digits until the end. Multiplying the already rounded 111,000 by $\cos(37.36^\circ)$ gives about 88,200, a worse estimate than 88,479.

On the WGS 84 ellipsoid at this latitude, one degree of latitude spans about 110,995 m and one degree of longitude about 88,590 m. The lesson factors sit within roughly 0.1% of both, or about 10 cm over a 100 m walk, while a phone position is uncertain by a few meters. Rounding is therefore not automatically a loss of accuracy: 111,000 is closer to the ellipsoidal value than the unrounded spherical 111,317 is.

The model is not the largest source of error in this lab. Report that way: quote distances to the precision your measurement supports, not to the precision the conversion factor could carry.

Combine short perpendicular changes

For nearby endpoints, use a representative local latitude and estimate:

$d \approx \sqrt{[(\Delta\text{north})^2 + (\Delta\text{east})^2]}$.

This is local endpoint separation. For a route, add the lengths of successive short segments. Sparse samples can miss turns; noisy samples can add spurious distance.

Optional mathematics appendix

What a decimal place is worth

Read the decimal places

Because the two factors differ, the same coordinate step is worth different distances on the two axes. Near Merced:

Coordinate step	Northward (Δ latitude)	Eastward (Δ longitude)
0.1°	11.1 km	8.85 km
0.01°	1.11 km	885 m
0.001°	111 m	88.5 m
0.0001°	11.1 m	8.85 m
0.00001°	1.11 m	0.885 m

The fifth decimal place is worth about a meter, which is already smaller than a phone’s typical horizontal accuracy. Digits beyond it describe the number, not the location.

Know when to change the model

A local map with fixed Merced factors stays accurate only while the latitude barely changes. Move 1 km north and the longitude factor shifts by about 0.01%. Move 100 km north, roughly 0.9° of latitude, and it shifts by more than 1%, which is ten times the error in the factors themselves.

Curvature is not what breaks first. Treating a few hundred meters of surface as flat costs less than a micrometer. The error you actually carry is in the conversion factors, and it grows once you travel far enough north or south to change the latitude.

A single flat map with fixed Merced factors cannot accurately represent a journey across the globe. Great-circle calculations, such as the haversine formula, account for spherical curvature; ellipsoidal geodesic calculations also account for Earth’s flattened shape. Except along the equator, a latitude-circle arc is not the shortest global route between two points.

Radius reference: NASA/JPL, Planetary Physical Parameters, which lists Earth’s equatorial radius as 6,378.1366 km and its mean radius as 6,371.0084 km. The spherical radius used here is a rounded modeling choice.
https://ssd.jpl.nasa.gov/planets/phys_par.html